



UNIVERSIDAD TÉCNICA
FEDERICO SANTA MARÍA

DEPARTAMENTO
DE INFORMÁTICA

The Future of Computing: A Survey of Quantum Computing, Quantum Machine Learning, and Quantum Cryptography

Felipe Cisternas Alvarez
Jean-Pierre Villacura

Introduction to Quantum Computing

Professor Mauricio Solar

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Executive Summary

The following report corresponds to a comprehensive investigation of the state of the art in the article 'An extensive review of Quantum Computers'.

We chose to consider topics relevant to quantum computing that the original article does not consider, such as machine learning, and the deepening of other issues related to cybersecurity.

We will introduce the reader to the basic concepts of quantum computing so that they have a useful context and can easily understand the terms mentioned in the investigation. We will analyze different recent publications, and we will give a summary of the contributions made.

Finally we conclude with the analysis of the bibliography, the research centers, the current state of the art, surprising results and personal conclusions.

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1 Introduction

With certainty it can be stated that today's computing is much faster than the computing of 50 years ago, the computers of that time were large, heavy, with a capacity and processing speed very limited to what is the standard now. We could consider quantum computing to be in this same state, as an emerging technology that is still expensive, bulky and with a lot of research potential.

The theory of quantum computing points out that its processing speed can be much faster than even the fastest supercomputer today. Examples such as Shor's algorithm with its potential ability to factor large prime numbers in a matter of seconds, as opposed to the thousands of years that classical computing could take, are considered signs of the advances and development that is to come with quantum computing.

This paper explores a range of subjects concerning quantum computing, including quantum computers and technologies. It is structured to provide readers with a comprehensive understanding, starting from the basics of quantum computing and progressing to cover a wide range of proposed models for quantum computers. Additionally, the paper delves into the future prospects and developments of the fields **Quantum Machine Learning** and **Quantum cryptography**, highlighting the immense potential of quantum computing and discussing current advancements. The structure of the paper includes the following:

1. Introduction
2. Brief overview of Quantum Computing
 - 2.1. Quantum Computing
 - 2.2. Quantum Computers and Technologies
 - 2.3. Quantum Data
 - 2.4. Quantum Gates
 - 2.5. Noise
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3. State of the Art
 - 3.1. Methodology
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2 Brief overview of Quantum Computing

2.1 Quantum Computing

Quantum computing relies on properties of quantum mechanics to compute problems that would be out of reach for classical computers. A quantum computer uses qubits. Qubits are like regular bits in a computer, but with the added ability to be put into a superposition and share entanglement with one another (S, Singh, and N (2022)).

A quantum computer works using quantum principles. Quantum principles require a new dictionary of terms to be fully understood, terms that include superposition, entanglement, and decoherence. Let's understand these principles below.

- **Superposition:** Superposition states that, much like waves in classical physics, you can add two or more quantum states and the result will be another valid quantum state. Conversely, you can also represent every quantum state as a sum of two or more other distinct states. This superposition of qubits gives quantum computers their inherent parallelism, allowing them to process millions of operations simultaneously.
- **Entanglement:** Quantum entanglement occurs when two systems link so closely that knowledge about one gives you immediate knowledge about the other, no matter how far apart they are. Quantum processors can draw conclusions about one particle by measuring another one, Quantum entanglement allows quantum computers to solve complex problems faster. When a quantum state is measured, the wavefunction collapses and you measure the state as either a zero or a one. In this known or deterministic state, the qubit acts as a classical bit. Entanglement is the ability of qubits to correlate their state with other qubits.
- **Decoherence:** Decoherence is the loss of the quantum state in a qubit. Environmental factors, like radiation, can cause the quantum state of the qubits to collapse. A large engineering challenge in constructing a quantum computer is designing the various features that attempt to delay decoherence of the state, such as building specialty structures that shield the qubits from external fields.

The current state of quantum computing is referred to as the noisy intermediate-scale quantum (NISQ) era (Brooks (2019)), characterized by quantum processors containing 50–100 qubits which are not yet advanced enough for fault-tolerance or large enough to achieve quantum supremacy, the term NISQ was coined by John Preskill in 2018 (Preskill (2018)). These processors, which are sensitive to their environment (noisy) and prone to quantum decoherence, are not yet capable of continuous quantum error correction. This intermediate-scale is defined by the quantum volume, which is based on the moderate number of qubits and gate fidelity.

Classical computers perform deterministic classical operations or can emulate probabilistic processes using sampling methods. By harnessing superposition and en-

tanglement, quantum computers can perform quantum operations that are difficult to emulate at scale with classical computers. Ideas for leveraging NISQ quantum computing include optimization, quantum simulation, cryptography, and machine learning.

Notably, quantum computers are believed to be able to solve many problems quickly that no classical computer could solve in any feasible amount of time—a feat known as *quantum supremacy*.

2.2 Quantum computers and technologies

A quantum computer is a computer that exploits quantum mechanical phenomena. At small scales, physical matter exhibits properties of both particles and waves, and quantum computing leverages this behavior using specialized hardware. Classical physics cannot explain the operation of these quantum devices, and a scalable quantum computer could perform some calculations exponentially faster than any modern *classical*.

No one has shown the best way to build a fault-tolerant quantum computer, and multiple companies and research groups are investigating different types of qubits. We give a brief example of some of these qubit technologies below.

- **Gate-based ion trap processors:** Trapped ion quantum computers implement qubits using electronic states of charged atoms called ions. The ions are confined and suspended above the microfabricated trap using electromagnetic fields. Trapped-ion based systems apply quantum gates using lasers to manipulate the electronic state of the ion (Whitfield, Yang, Wang, Heath, and Harrison (2022)). Trapped ion qubits use atoms that come from nature, rather than manufacturing the qubits synthetically.
- **Gate-based superconducting processors:** Superconducting quantum computing is an implementation of a quantum computer in superconducting electronic circuits. Superconducting qubits are built with superconducting electric circuits that operate at cryogenic temperatures.
- **Photonic processors:** A quantum photonic processor is a device that manipulates light for computations. Photonic quantum computers use quantum light sources that emit squeezed-light pulses, with qubit equivalents that correspond to modes of a continuous operator, such as position or momentum.
- **Neutral atom processors:** Neutral atom qubit technology is similar to trapped ion technology. However, it uses light instead of electromagnetic forces to trap the qubit and hold it in position. The atoms are not charged and the circuits can operate at room temperatures
- **Rydberg atom processors:** A Rydberg atom is an excited atom with one or more electrons that are further away from the nucleus, on average. Rydberg

atoms have a number of peculiar properties including an exaggerated response to electric and magnetic fields, and long life. When used as qubits, they offer strong and controllable atomic interactions that you can tune by selecting different states.

- **Quantum annealers:** Quantum annealing uses a physical process to place a quantum system's qubits in an absolute energy minimum. From there, the hardware gently alters the system's configuration so that its energy landscape reflects the problem that needs to be solved. The advantage of quantum annealers is that the number of qubits can be much larger than those available in a gate-based system. However, their use is limited to specific cases only.

2.3 Quantum Data

Quantum data is any data source that occurs in a natural or artificial quantum system. Quantum data exhibits superposition and entanglement, leading to joint probability distributions that could require an exponential amount of classical computational resources to represent or store. The quantum supremacy experiment showed it is possible to sample from an extremely complex joint probability distribution of 2^{53} Hilbert space.

The qubit serves as the basic unit of quantum information. It represents a two-state system, just like a classical bit, except that it can exist in a superposition of its two states. In one sense, a superposition is like a probability distribution over the two values. However, a quantum computation can be influenced by both values at once, inexplicable by either state individually. In this sense, a *superposed* qubit stores both values simultaneously. When measuring a qubit, the result is a probabilistic output of a classical bit. If a quantum computer manipulates the qubit in a particular way, wave interference effects can amplify the desired measurement results.

The quantum data generated by NISQ processors are noisy and typically entangled just before the measurement occurs. Heuristic machine learning techniques can create models that maximize extraction of useful classical information from noisy entangled data.

The following are examples of quantum data that can be generated or simulated on a quantum device:

- **Chemical simulation:** Extract information about chemical structures and dynamics with potential applications to material science, computational chemistry, computational biology, and drug discovery.
- **Quantum matter simulation:** Model and design high temperature superconductivity or other exotic states of matter which exhibits many-body quantum effects.
- **Quantum control:** Hybrid quantum-classical models can be variationally trained to perform optimal open or closed-loop control, calibration, and error

mitigation. This includes error detection and correction strategies for quantum devices and quantum processors.

- **Quantum communication networks:** Use machine learning to discriminate among non-orthogonal quantum states, with application to design and construction of structured quantum repeaters, quantum receivers, and purification units.
- **Quantum metrology:** Quantum-enhanced high precision measurements such as quantum sensing and quantum imaging are inherently done on probes that are small-scale quantum devices and could be designed or improved by variational quantum models.

2.4 Quantum Gates

The state of qubits can be manipulated by applying quantum logic gates, analogous to how classical bits can be manipulated with classical logic gates. Unlike many classical logic gates, quantum logic gates are reversible. Quantum logic gates are represented by unitary matrices, a gate which acts on n qubits is represented by $2^n \times 2^n$ unitary matrix.

Quantum states are typically represented by *kets*, from a notation know as **bra-ket**, the vector representation of a single qubit is

$$|a\rangle = v_0|0\rangle + v_1|1\rangle \rightarrow \begin{bmatrix} v_0 \\ v_1 \end{bmatrix}$$

Here v_0 and v_1 are the complex probability amplitudes of the qubit, these values determine the probability of measuring a 0 or a 1, when measuring the state of the qubit. The value zero is represented by the ket

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and the value one is represented by the ket

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The tensor product denoted by the symbol \otimes , is used to combine quantum states. The action of the gate on a specific quantum state is found by multiplying the vector $|\phi_1\rangle$ which represents the state by the matrix U representing the gate, thus the result is a new quantum state $|\phi_2\rangle$

$$U|\phi_1\rangle = |\phi_2\rangle$$

There's exist many number of quantum gates, below we are going to review some of the most often used in the literature:

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- **NOT Gate:** This gate is widely known as X-Pauli Gate, as this particular quantum gate transforms the existing state of the qubit to be rotated around the X-axis. As the name suggests, the NOT gate would convert a qubit from its initial state to its complement state. This quantum gate is represented by a matrix:

$$X := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$X|0\rangle = |1\rangle \text{ and } X|1\rangle = |0\rangle.$$

- **Y-Pauli Gate:** The Y-Pauli gates are capable of rotating the input qubit around the Y-axis. This quantum gate is represented by a matrix:

$$Y := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Y|0\rangle = i|1\rangle \text{ and } Y|1\rangle = -i|0\rangle.$$

- **Z-Pauli Gate:** The Z-Pauli or phase flip gate are capable of rotating the input qubit around the Z-axis. This quantum gate is represented by a matrix:

$$Z := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli Z leaves the basis state $|0\rangle$ unchanged and maps $|1\rangle$ to $-|1\rangle$

$$Z|0\rangle = |0\rangle \text{ and } Z|1\rangle = -|1\rangle.$$

- **Controlled NOT Gate:** the controlled NOT (CNOT) gate acts on 2 (or more) qubits, and performs the NOT operation on the second (or more) qubit only when the first qubit is $|1\rangle$, this gate is represented by the Hermitian unitary matrix:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$CNOT|00\rangle = |00\rangle$$

$$CNOT|01\rangle = |01\rangle$$

$$CNOT|10\rangle = |11\rangle$$

$$CNOT|11\rangle = |10\rangle$$

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- **Hadamard gate:** The Hadamard gate, acts on a single qubit and creates an equal superposition states given a basis state, The Hadamard gate performs a rotation of π about the axis $(\hat{x} + \hat{z})/\sqrt{2}$ at the *Bloch Sphere*, this gate is represented by the Hadamard matrix:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H|0\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}} \text{ and } H|1\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}}.$$

2.5 Noise

Noise is present in modern day quantum computers. Qubits are susceptible to interference from the surrounding environment, imperfect fabrication, TLS and sometimes even gamma rays. Until large scale error correction is reached, the algorithms of today must be able to remain functional in the presence of noise. This makes testing algorithms under noise an important step for validating quantum algorithms and quantum models.

2.6 Quantum error correction

Quantum error correction (QEC) is used in quantum computing to protect quantum information from errors due to decoherence and other quantum noise. Traditional error correction employs over repetition. The repetition code is the simplest but most inefficient way. The idea is to store the information multiple times and take a larger vote in the event that these copies are later found to differ. Copying quantum information is not possible due to the no-cloning theorem. This theorem seems to present an obstacle to formulating a theory of quantum error correction, nevertheless, it is conceivable to transfer the logical information of a single qubit to a highly entangled state of several physical qubits (Raussendorf (2012)).

2.7 Quantum Cybersecurity

2.7.1 Quantum Cryptography

Cryptography has had an important development since 1975 with the establishment of the DES algorithm for file encryption while computing was emerging. Since then, several algorithms have been developed to fulfill this cryptographic function, the best known being RSA and AES.

With the emergence of quantum computing, several researchers comment on the risk that the rise of this paradigm may threaten encryption algorithms based on classical computing, which are considered secure due to the amount of time it takes to test all combinations, easily 50 years using supercomputers. Quantum computing threatens the security of these algorithms by being able to perform calculations much

faster. Being able to solve operations that in the classical paradigm would take about 50 years using supercomputers in a matter of seconds.

The state of the art of quantum computing proposes modifications to algorithms RSA and AES.

- Ko and Jung (2021) proposes a modified AES Algorithm comparing different methods of random number generation, resulting in the use of **Quantum Random Walk** the best encryption performance. It is proposed the modifying the Shift row operation introducing random movements using QRW, making it difficult to predict the correct order during the decryption process. This adds an additional layer of complexity and makes attempts to decrypt encrypted information difficult without proper knowledge of the correct sequence.
- (Bernstein, Heninger, Lou, and Valenta (2017)) proposes that the little investigation about RSA algorithm in Quantum Computing is due to actual limits of Shor's Algorithm. They propose a Quantum ring algorithm: GEECM, using pre-quantum algorithm to find small primers and accelerate it with quantum techniques.

2.7.2 Quantum Key Distribution

This topic is closely related to cryptography, since the security of the data depends on the transmission of the key previously generated by a cryptographic algorithm. There are dangers associated with the transmission of the key that were partly solved with the introduction of the asymmetric key (whose most famous algorithm is RSA).

Quantum computing proposes new solutions in this aspect by being able to transmit the same key to two recipients (Alice and Bob), with a very high certainty of corroborating that there is no third person obtaining this key. This is possible due to quantum properties such as entanglement and non-cloning. With entanglement it is possible to know the state of both photons (direction of spin) and non-cloning allows to detect if there is any intruder in the system, since it would yield a common key different from Alice and Bob.

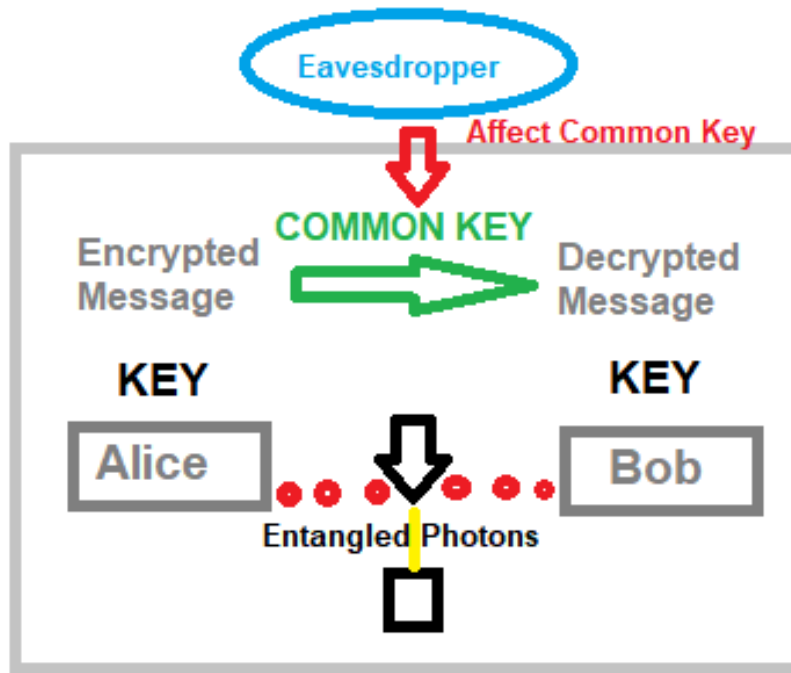


Figure 1: Quantum Key Distribution (Source Diagram: Author's own work)

Since Alice and Bob have the same common key, Alice can encrypt her message and send it to Bob, who has the key to decrypt and therefore read the message. We assume noise-free Channel for this situation.

Quantum computing can also contribute to the 'one-time-pad' problem of classical computing by providing random keys due to the Heisenberg uncertainty principle. Note that in this type of problem security depends to some degree on the randomness of the key.

2.8 Quantum Machine Learning

One of the most successful technologies of this century is machine learning (ML), Machine learning is a subset of artificial intelligence (AI) that focuses on developing algorithms and models that enable computers to learn from data and make predictions or decisions without being explicitly programmed.

Like other classical theories, ML and learning theory can in fact be embedded into the quantum mechanical formalism. Formally speaking, this embedding leads to the field known as Quantum Machine Learning which aims to understand the ultimate limits of data analysis allowed by the laws of physics. While there are similarities between classical and quantum ML, there are also some differences. Because QML employs quantum computers, noise from these computers can be a major issue.

In ML we have different paradigms that also applies to QML:

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- Supervised Learning (Task-based)
 - Unsupervised Learning (Data-based)
 - Reinforcement Learning (Reward-based)

and there is a bunch of algorithms of QML being researched:

- Variational Quantum Algorithms (VQAs)
- Quantum Neural Networks (QNN)
- Quantum Kernels

2.8.1 Quantum Neural Networks

A quantum neural network is used to describe a parameterized quantum computational model that is best executed on a quantum computer. This term is often interchangeable with parameterized quantum circuit (PQC). These involve a sequence of unitary gates acting on the quantum data states $|\psi_j\rangle$, some of which have free parameters θ that will be trained to solve the problem.

QNNs are employed in all three QML paradigms mentioned above. For instance, in a supervised classification task, the goal of the QNN is to map the states in different classes to distinguishable regions of the Hilbert space, in the unsupervised learning scenario of a clustering task is mapped onto a MAXCUT problem (Otterbach et al. (2017)) and solved by training a QNN to maximize distance between classes, finally, in the reinforced learning task of a QNN can be used as the Q-function approximator (Skolik, Jerbi, and Dunjko (2022)), which can be used to determine the best action for a learning agent given its current state.

As in classical neural networks, there are different types of networks such as convolutional networks, recurrent networks, etc. their quantum variants have been researched, such as quantum convolutional neural networks (Cong, Choi, and Lukin (2019)) and quantum recurrent neural networks (Bausch (2020)).

2.8.2 Quantum Kernels

In machine learning, A kernel is a function that defines the similarity or distance between pairs of data points in a high-dimensional feature space. Quantum kernel methods consider the computation of kernel functions using quantum computers.

There are many possible implementations. For example considered a reproducing kernel Hilbert space equal to the quantum state space, which is finite dimensional, in simpler terms, we can think of the quantum state space as a finite-dimensional space (Schuld (2021)). By using this approach, we can calculate kernel functions within this finite-dimensional space.

Another approach involves studying a reproducing kernel Hilbert space that is infinite-dimensional, in this case, we are transforming classical vectors (which represent data points) using a quantum computer, the quantum computer helps us map these classical vectors into infinite-dimensional vectors, an infinite-dimensional space allows for more complex representations and calculations.

2.8.3 Variational Quantum Algorithms

Variational Quantum Algorithms (VQAs) are a hybrid quantum-classical optimization algorithm in which an objective function is evaluated by quantum computation, and the parameters of this function are updated using classical optimization methods (Cerezo et al. (2021)).

The variational method in quantum theory is a classical method for finding low energy states of a quantum system. The idea of this method is that one defines a wave function (called an ansatz) as a function of some parameters, and then one finds the values of these parameters that minimize the expectation value of the energy.

It has been realized that quantum computers can mimic the classical technique and that a quantum computer does so with certain advantages (Peruzzo et al. (2014), Wecker, Hastings, and Troyer (2015)), when one applies the classical variational method to a system of n qubits, an exponential number of complex numbers is necessary to generically represent the wave function of the system. However, with a quantum computer, one can directly produce this state using a parameterized quantum circuit with less than exponential parameters.

2.8.4 Inductive Bias

Inductive bias means that any model, can only represent a subset of all possible functions and is naturally inclined towards certain types of functions. These functions relate the input features to the output predictions.

Inductive bias encompasses the assumptions and restrictions made in the model design and optimization process, shaping the search space for potential models. The choice of model parameterization or embedding, as well as techniques like regularization and learning rate modulation, contribute to the inductive bias.

To achieve quantum advantage with Quantum Machine Learning (QML), we aim for QML models that have an inductive bias that is difficult to simulate efficiently using classical models. Recent research has shown that it is possible to construct quantum kernels with this property (Kübler, Buchholz, and Schölkopf (2021)), although there are some complexities regarding their trainability.

3 State of the Art

3.1 Methodology

To gather new information on Quantum Cybersecurity it was used an method based on paper 'The Bibliographic Search in ten stages ' Amezcua (2015)

- **Relevant Topic:** Quantum Cybersecurity , **Format:** Investigation and State of art. **Specialized Authors:** Abd El-Latif, Ahmed. , **Time Frame:** 2021-2023.
- **Keywords:** Quantum, post-quantum, Cybersecurity, encryption, Key-Distribution, Authentication, Digital signature, IoT.
- **BDB:** Web of Science, Springerlink.

Meanwhile, to gather new information on Quantum Machine Learning, it was used the Snowball Methodology, reading the citations of the most recent papers on Quantum Machine Learning.

3.2 Description of New Works and Research

- **Development of Cybersecurity Technology and Algorithm Based on Quantum Computing** (Ko and Jung (2021))

In this work, the authors propose a modified AES algorithm and use quantum computing to encrypt/decrypt AES image files using IBM Qiskit for performance evaluation. They show that AES algorithm can be implemented using quantum gates and suggest that AES be implemented with random number generation.

Advanced Encryption System (AES) is combined with the use of random number generation in the process. In the traditional implementation of the AES algorithm, the Shift Row operation moves the data to align them at certain encryption steps. Since the decryption process can reverse the order of these steps, it becomes predictable. To address this vulnerability, the author suggests modifying the performance of the Shift Row operation to introduce random movements using Quantum Random Walk, making it difficult to predict the correct order during the decryption process, achieving greater security than classical approach.

- **Quantum Cryptography for the future internet and security Analysis** Tianqi Zhou (2018) In this work, the authors focus on analyzing characteristics of the quantum cryptography and exploring of the advantages of it in the future internet. They analyze the Quantum Key Distribution protocol in the noise-free channel by making measurements of different variables. Probability of

the eavesdropper being detected v/s Number of photons measured in a noise-free Channel and 30 % noise. Also analyzes the probabilities of errors in the receiver v/s Probability of eavesdropper to eavesdrop on the channel.

- **A comprehensive Tutorial on Cybersecurity in Quantum Computing Paradigm** (Uttam Ghosh (2023))

In this work, the authors make a contribution in the state of art of Cybersecurity from wide perspectives.

They give an overview of Quantum Computing and how it can affect cybersecurity issues. Also demonstrate solutions in Quantum computing to problems in classical computing paradigm related to cybersecurity, and relates how Quantum computing could be used in the future to make cybersecurity solutions better.

- **Post-Quantum RSA** Bernstein et al. (2017)

In this work, the authors make a contribution proposing parameters and changes to RSA to make Key-Generation, encrypt and decryption, signatures and verification feasible in actual computing and, at the same time, protected against quantum computing attacks.

Proposes a new Quantum Algorithm to generate factor numbers, GEECM faster than Shor and algorithms of classic paradigm and a new .

- **Quantum agents in the Gym: a variational quantum algorithm for deep Q-learning** (Skolik et al. (2022))

In this work, the authors introduce a new training method for parametrized quantum circuits (PQCs) that can be used to solve RL tasks for discrete and continuous state spaces based on the deep Q-learning algorithm.

They adapt the DQN algorithm to use a PQC as its Q-function approximator instead of a NN, for this they use a hardware-efficient ansatz, a target network, an ϵ -greedy policy to determine the agent's next action and experience replay to draw samples for training the Q-Network PQC. The Q-Network then is $U_\theta(s)$ parametrized by θ and the target network PQC is $\hat{U}_{\theta_\delta}(s)$, where θ_δ is a snapshot of the parameters θ which is taken after fixed intervals of episodes δ and the circuit is otherwise identical to that $U_\theta(s)$.

Depending on the state the authors distinguish between two different types of space states: *Discrete* and *Continuous*.

The Q-Values of the quantum agent are computed as the expectation values of a PQC that is fed a state s as:

$$Q(s, a) = \langle 0^{\otimes n} | U_\theta^\dagger(s) O_a U_\theta(s) | 0^{\otimes n} \rangle$$

where O_a is an observable and n the number of qubits, and the model outputs a vector including Q-values for each possible O_a .

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- **Out-of-distribution generalization for learning quantum dynamics** (Caro et al. (2022))

In this work, the authors demonstrate the out-of-distribution generalization for the task of learning in Quantum Machine Learning, where the training and testing data are drawn from a different distribution.

The authors consider the QML task of learning an unknown n -qubit unitary $U \in \mathcal{U}(\mathbb{C}^{2^n})$. The goal is to use training states to optimize the classical parameters α of $V(\alpha)$, an n -qubit unitary QNN, such that for the optimized parameters α_{opt} , $V(\alpha_{opt})$ well predicts the action of U on previously unseen test states. The prediction performance of the trained QNN $V(\alpha_{opt})$ can be quantified in terms of the average distance between the output state predicted by $V(\alpha_{opt})$ and the true output state determined by U .

They provide numerical evidence to support analytical results showing that out-of-distribution generalization is possible for the learning of quantum dynamics, they focused on the task of learning the parameters of an unknown target Hamiltonian by studying the evolution of product states under it.

The authors work establishes that for learning unitaries, QNNs trained on quantum data enjoy out-of-distribution generalization between some physically relevant distributions if the training data size is roughly the number of trainable gates.

- **Operator Sampling for Shot-frugal Optimization in Variational Algorithms** (Arrasmith, Cincio, Somma, and Coles (2020))

In this work, the authors propose a new strategy for reducing the number of measurements in variational quantum-classical algorithms (VQCAs) needed for convergence.

VQCAs efficiently evaluate a cost function on a quantum computer while optimizing the cost value using a classical computer. Certain issues arise in VQCAs that are not common in classical algorithms, implying that standard off-the-shelf classical optimizers may not be best suited to VQCAs. For example, multiple runs of quantum circuits are required to reduce the effects of shot noise on cost evaluation, An additional complication is that quantum hardware noise flattens the training landscape.

The authors have recently investigated shot-frugal gradient descent for VQCAs, introduced an optimizer, called iCANS (individual Coupled Adaptive Number of Shots), which outperformed off-the-shelf classical optimizers such as Adam for variational quantum compiling and variational quantum eigensolver (VQE) tasks. The key feature of iCANS is that it maximizes the expected gain per shot by frugally adapting the shot noise for each individual partial derivative.

In VQE and other VQCAs, it is common to express the cost function $C = \langle H \rangle$ as the expectation value of a Hamiltonian H that is expanded as a weighted

sum of directly measurable operators $\{h_i\}_i$:

$$H = \sum_{i=1}^N c_i h_i$$

then C is computed from estimations of each expectation $\langle h_i \rangle$, which is obtained from many shots. The authors proposal is to randomly assign shots to the h_i operators according to a weighted probability distribution proportional to $|c_i|$, they prove that this leads to an unbiased estimator of the cost C , even when the number of shots is extremely small like a single shot. This allows one to unlock a level of shot-frugality for unbiased estimation that simply cannot be accessed without operator sampling. In addition, the randomness associated with operator sampling can provide a means to escape from local minima of C .

A combination of the new sampling strategy with iCANS leads to the main result, which is an improved optimizer for VQCs that they call Rosalin (Random Operator Sampling for Adaptive Learning with Individual Number of shots). Rosalin retains the crucial feature of maximizing the expected gain per shot. the authors analyze the potential of Rosalin by applying it to VQE for three molecules, namely H_2 , LiH , and BeH_2 , and compare its performance with that of other optimizers. In cases with more than a few terms in the Hamiltonian, Rosalin outperforms all other optimizer and sampling strategy combinations considered.

- **Quantum natural gradient generalised to noisy and non-unitary circuits** (Koczor and Benjamin (2022))

In this work, the authors generalise a quantum natural gradient to consider arbitrary quantum states to significantly outperform other variational quantum algorithms.

Quantum Fisher information in the context of general variational quantum circuits is a measure that quantifies how much and in what way changing parameters in a quantum circuit affects the underlying quantum state.

The aim of the authors is to minimise the expectation value $E(\underline{\theta}) = Tr[\rho(\underline{\theta})\mathcal{H}]$ of a Hermitian observable \mathcal{H} over the parameters $\underline{\theta}$ using a variational quantum circuit that depends on these parameters, this circuit produces the quantum states $\rho(\underline{\theta}) = \Phi(\underline{\theta})\rho_0$ via mapping, and might involve non-unitary transformations due to experimental imperfections or indeed intentional non-unitary elements, such as measurements.

The authors propose a natural gradient update rule, where the quantum Fisher information matrix \mathbf{F}_q corrects the gradient vector g_k to account for the dependent and non-uniform effect of the parameters on an arbitrary quantum state $\rho(\underline{\theta})$ mixed or pure. their method also applies to infinite-dimensional quantum states as continuous-variable systems.

The natural gradient descent proposed by the authors in principle allows for improvements relative to imaginary time evolution and the pure-state variant of natural gradient. First even when the objective function is generated by an observable as $E(\underline{\theta}) = \text{Tr}[\rho(\underline{\theta})\mathcal{H}]$, their approach allows for general non-unitary elements as Completely positive trace-preserving (CPTP) maps which in principle enable the manipulation of exponentially more degrees of freedom. Second the expected value $E(\underline{\theta}) := \text{Tr}[\rho(\underline{\theta})\mathcal{H}]$ is a mapping that is linear in quantum state, their results shown that are well-defined for more general objective functions and its convergence is guaranteed even in the presence of shot noise.

When compared to previous studies, the new approach has the advantage that it explicitly takes into account imperfections of the variational quantum circuit.

- **Training Quantum Embedding Kernels on Near-Term Quantum Computers** (Hubregtsen et al. (2022))

In this work, the authors provide an accessible introduction to quantum embedding kernels (QEK) and then analyze the practical issues arising when realizing them on a noisy near-term quantum computer.

QEK are a subclass of quantum kernel methods where a PQC is used to embed datapoints into the Hilbert space of quantum states. QEKs have certain appealing properties that make them attractive for use, like they limited depth does not require long coherence times, another strong point is that noisy PQCs still lead to well-defined QEKs.

The authors propose a series of improvements, first to use the *kernel-target alignment* as a cost function to train parameters of the QEK to increase its performance on particular datasets, second they propose a mitigation strategy tailored for the QEKs that exploits the kernel’s definition to infer the underlying noise levels, lastly they propose a strategy for alleviate the influence of noise on the kernel matrix based on a semi-definite program.

The quantum embedding kernel is defined as the inner product between quantum states, which is given by the overlap

$$k(\mathbf{x}, \mathbf{x}') = |\langle \phi(\mathbf{x}') | \phi(\mathbf{x}) \rangle|^2$$

Associated to the quantum feature map $|\phi(\mathbf{x})\rangle$, but we are not able to avoid noise, which means that we cannot assume that the embedded quantum state is pure, then the quantum embedding is realized by a data-dependent density matrix $\rho(\mathbf{x})$ which for pure states reduces to $\rho(\mathbf{x}) = |\phi(\mathbf{x})\rangle\langle\phi(\mathbf{x})|$, with this modification the inner product is given by

$$k(\mathbf{x}, \mathbf{x}') = \text{Tr}\{\rho(\mathbf{x})\rho(\mathbf{x}')\}$$

This inner product is also know as the Hilbert-Schmidt inner product for matrices. In summary, any quantum feature map induces a QEK. We can use this

kernel as a subroutine in a classical kernel method, for example the SVM, which yields a hybrid quantum-classical approach.

To be able to use QEKs in this way, is needed to evaluate the overlap of two quantum states on near-term hardware, there are a number of advanced algorithms to estimate the overlap of two quantum states . All these algorithms work for arbitrary states, and so they are agnostic to how the states were obtained by necessity. By exploiting the structure and specifics of QEKs, though. The authors propose a better ways to do this overlap, for unitary quantum embeddings they construct the adjoint of the data-encoding circuit $U^\dagger(\mathbf{x})$, another approach proposed is the SWAP *test*, the SWAP test is based on the SWAP *trick*, a mathematical gimmick that allow to transform the product of the density matrices into a tensor product.

Finally the authors have performed various numerical experiments that showed improvement in classification accuracy after training. They have also investigated noise mitigation techniques and proposed device noise mitigation techniques specific for kernel matrices and combined them with regularization methods. Lastly they tested a large set of combinations, both on simulated depolarizing noise as well as on data from a real quantum processing unit

3.3 Comparative analysis of the latest advances

Authors	contribution made	Comparative advantage
Andrea Skolik Sofiene Jerbi Vedran Dunjko	New training method for PQCs that can be used to solve Reinforcement learning tasks for discrete and continuous spaces based on the deep Q-learning algorithm	Training method for discrete and continuous state spaces for quantum circuits
Mathias Caro Hsin-Yuan Huang Nicholas Ezzel Joe Gibbs Andrew Sornborger Lukasz Cincio Patrick Coles Zoe Holmes	Demonstration the Out-of-Distribution generalization, for the task of learning in Quantum Machine learning where the training and testing data are drawn from different distributions	Ability to extrapolate from training data to unseen data with the potential of Quantum Machine Learning methods to outperform classical Machine Learning
Andrew Arrasmith Lukasz Cincio Rolando Somma Patrick Coles	New strategy for reducing the number of measurements with an adaptive optimizer to construct an improved optimizer called Rosalin that implements stochastic gradient descent while adapting the shot noise for each partial derivative and randomly assigning the shots according to a weighted distribution.	Rosalin outperforms other optimizers in the task to find the ground states of molecules H_2 , LiH , and BeH_2 without and with quantum hardware noise
Bálint Koczor Simon Benjamin	generalization of quantum natural gradient to consider arbitrary quantum states via completely positive maps, thus the circuits can incorporate both imperfect unitary gates and fundamentally non-unitary operations such as measurements	demonstration in numerical simulations of noisy quantum circuits the practicality of the new approach and confirm it can significantly outperform other variational techniques.
Thomas Hubregtzen David Wierichs Elies Gil-Fuster Peter-Jan Dereks Paul Faehrmann Johannes Meyer	An accessible introduction to quantum embedding kernels, a analysis of the practical issues arising when realizing them on a noisy near-term quantum computer, and a strategy to mitigate these detrimental effects which is tailored to quantum embedding kernels	Improvement in classification accuracy after training, noise mitigation techniques and regularization methods for specific kernel matrices.
Kyung-Kyu Ko Eun-SUng Jung	Propose of AES Algorithm for Quantum Computing with improved Security using Quantum Random Walk.	Propose of Quantum version of AES algorithm with improvement against Quantum attacks
Tianqi Zhou Jian Shen Xiong Li	Explication of Quantum Key Distribution and experiments with Quantum Noise	State of art about Quantum Key Distribution and experiments with eavesdropper
Daniel Bernstein Nadia Heninger	Propose parameters and changes to RSA, on Quantum Computing, to make feasible in actual.	Proposes a GEECM, faster algorithm than Shor and experiments with eavesdropper
Utham Ghosh Debashi Das Pushpita Shatterje	Give an overview of quantum computing related to Cybersecurity presenting several Quantum solutions and show how can be used in future to make the area better than now.	Proposes a state of art of Quantum attacks, and existing Quantum-based approaches for Cybersecurity.

Table 1: Comparative Table

3.4 Bibliographic Discussion

From the documentation consulted about the problems and the context of development of the search, it has been possible to delve into the new contributions and their functionalities.

3.4.1 Similarities and Differences

We have noticed that in the quantum machine learning field, researchers opted for different algorithms competing against each other to see which one gives the best results, quantum neural networks vs. quantum variational algorithms vs. quantum kernels, each one with its own pros and cons, it will be necessary to observe how these algorithms evolve with the passage of time and the advancement of technology.

3.4.2 Research groups and Institutions

It is necessary to highlight and thank the work of these research groups and their institutions, without which this development would not be possible.

- Hongik University, Sejong, Korea.
- Jiangsu Engineering Center of Network Monitoring, China.
- Nanjing University of Information Science and Technology, Nanjing, China.
- Chinese Academy of Sciences, Beijing, China.
- Institute of Information Engineering, China.
- State Key Laboratory of Information Security, China.
- Hunan University of Science and Technology, Xiangtan, China.
- Institute of Electrical and Electronics Engineers (IEEE).
- University of Illinois, Chicago, United States.
- Technische Universiteit Eindhoven, The Netherlands.
- University of Pennsylvania, Philadelphia, United States.
- Leiden University, Leiden, The Netherlands.
- Volkswagen DataLab, Munich, Germany.
- University of Innsbruck, Innsbruck, Austria.
- Technical University of Munich, Garching, Germany.
- Munich Center for Quantum Science and Technology (MCQST), Munich, Germany.
- Freie Universitat Berlin, Germany.
- California Institute of Technology (Caltech), Pasadena, California, United States.
- Los Alamos National Laboratory, Los Alamos, New Mexico, United States.
- University of Southern California, Los Angeles, California, United States.
- University of Oxford, Oxford, United Kingdom.
- Quantum Motion, London, United Kingdom.
- University of Cologne, Cologne, Germany.
- University of Copenhagen, Copenhagen, Denmark.

3.5 State of the Art Timeline

TABLE 2 Timeline of Quantum Computing Major Advances

1970	• James Park articulates the no-cloning theorem (Park (1970)).
1973	• Alexander Holevo articulates the Holevo's Theorem and Charles H. Bennet shows that computation can be done reversibly(Bennett (1973)).
1980	• Paul Beinoff describes the first quantum mechanical computer model (Benioff (1980)), Tomasso Toffoli introduces the Toffoli Gate (Toffoli (1980)).
1985	• David Deutsch describes the first universal quantum computer.
1992	• David Deutsch and Richard Jozsa propose a computational problem that can be solved efficiently with the Deutsch–Jozsa algorithm on a quantum computer.
1993	• Dan Simon invents an oracle problem, for which a quantum computer would be exponentially faster than a conventional computer.
1994	• Peter Shor publishes the Shor's Algorithm.
1995	• Peter Shor proposes the first schemes for quantum error correction (Shor (1995)).
1996	• Lov Grover invents he quantum database search algorithm.
2000	• Arun K. Pati and Samuel L. Braunstein proved the quantum no-deleting theorem.
2001	• First execution of Shor's algorithm.
2003	• Implementation of the Deutsch–Jozsa algorithm on an quantum computer.
2006	• First 12 qubit quantum computer benchmarked.
2007	• D-Wave Systems demonstrates use of a 28-qubit quantum annealing computer.
2009	• First electronic quantum processor created.
2010	• Single-electron qubit developed.
2014	• Scientists transfer data by quantum teleportation over a distance of 3 meters with zero percent error rate (Pfaff et al. (2014)).
2017	• IBM unveils 17-qubit quantum computer.
2018	• Google announces the creation of a 72-qubit quantum chip.
2019	• IBM reveals its biggest quantum computer yet, consisting of 53 qubits.
2020	• Google engineers report the largest chemical simulation on a quantum computer.
2021	• IBM claims that it has created a new 127 quantum bit processor.
2022	• Researchers at Google Quantum AI Team Make Traversable Wormhole with a Quantum Computer.
2023	• researchers of Innsbruck have entangled two ions over a distance of 230 meters.

3.6 Expected or surprising results

While we were looking for information for our survey, we read amazing papers, where you would think you are reading science fiction or that it came out of a movie, but they are real scientific investigations like the publication in Nature, *Traversable wormhole dynamics on a quantum processor* (Jafferis et al. (2022)), We know that this paper caused some controversy and what we least want is to get into controversy, but we must not forget that it was an amazing experiment and discovery.

Another point that caught our attention was the rapid advance of quantum computing, it is a subject that is not heard as much as artificial intelligence or neural networks, but it is a field that is advancing by leaps and bounds. so we are happy to be able to contribute with this survey.

4 Conclusions

In this paper, we began with the fundamentals before moving on to cover quantum computing and related technology. Quantum Computing is still in its early stages, and building a functional and efficient computer with enough qubits takes years.

The QML domain should also target designing new quantum learning models that will observe patterns under quantum mechanics schemes, not classical statistical theory. This will provide an opportunity to explore new model architectures that might overcome classical machine learning limitations.

Quantum computers have the ability to simulate molecular behavior at a fundamental level, making them valuable for various industries. Automakers like Volkswagen and Daimler use quantum computers to analyze and improve the composition of electric vehicle batteries. Pharmaceutical companies also utilize quantum computers to study chemicals and explore new possibilities for medicine development. Quantum computing has the potential to revolutionize society, with its ability to solve optimization problems quickly by evaluating numerous solutions. Airbus employs quantum computers to determine fuel-efficient flight paths, while Volkswagen has developed a tool for optimizing bus and taxi routes to reduce traffic congestion. Some scientists believe that quantum computers could accelerate advancements in artificial intelligence. However, the full extent of quantum computing's potential may take many years to realize.

The development of post-quantum cryptography is crucial to mitigate the cybersecurity risks posed by quantum computing. Post-quantum cryptography refers to algorithms that are resistant to attacks from quantum computers. It not only improves database search capabilities but also addresses optimization problems in various business domains such as data analytics, logistics, and medical research.

Discovering better algorithms to work with quantum computing is still an open area of research. Finally, this study gives an overview of quantum machine learning, quantum cybersecurity and recent studies in quantum computations with its possible

applications. In future this work can be extended to provide a deeper view of quantum improvements implementation on real quantum machines.

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